

Numerical calculation of the temperature rise in ignition by hot wires

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With 6 figures in the text

The thermal conditions for a rapid rise in temperature in an explosive medium can in special cases be determined with satisfactory accuracy by analytical approximations to the solutions of the differential equations. Frank-Kamenetsky thus successfully estimated the critical temperature for explosion. He calculated the maximum wall temperature at which a stationary distribution of temperature is possible assuming decomposition according to the Arrhenius law and proportionality between the rates of decomposition and chemical heat production. The instantaneous rise in temperature which is required within a small volume, to obtain an explosion cannot, however, be estimated in this manner. This case is much more complicated and the only method available seems to be numerical integration with the aid of computers. In this way Cook (2) has calculated the time of ignition when a surface of an explosive stick is brought into contact with a hot bath kept at a constant temperature.

This article gives the results of a numerical calculation of the temperature development in the explosive medium around a wire, the temperature of which is suddenly raised by applying a certain amount of energy. The explosive medium is assumed to decompose according to the Arrhenius law, when the explosive heat is released.

The calculation has been carried out with the "Besk" (Stockholm) computer. The writers wish to express their thanks to Mr. G. Ehrling, Ph. L., who undertook the coding.

Symbols

- t the time, sec.
- r the distance to the axis of the wire, cm.
- a radius of the wire, cm.
- T the temperature, K .
- T_0 the temperature in the wire and charge when $t < 0$, K .
- T_1 the temperature of the wire when $t = 0$, K .
- A the frequency constant, s^{-1} .
- E activating energy, cal/mol.

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- R the gas constant, cal/mol K.
 Q heat of explosion, cal/g.
 n undecomposed fraction of the explosive medium.
 ρ_t density of the wire, g/cm³.
 c_t specific heat of the wire, cal/g C.
 ρ density of the explosive, g/cm³.
 c specific heat of the explosive, cal/g C.
 λ thermal conductivity of the explosive, cal/cm s C.
 U energy per unit length of wire necessary for explosion, cal/cm.

The temperature of the wire is assumed to be the same across the entire cross section. The error resulting from this simplification is in all probability slight and decreases with a diminishing wire radius. The terminal effects are also ignored, i.e. the wire is assumed to be infinitely long.

For the temperature around the wire we have the equations

$$c\rho \frac{\partial T}{\partial t} = -Q\rho \frac{\partial n}{\partial t} + \lambda \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right)$$

$$\frac{\partial n}{\partial t} = -nAe^{-E/RT}$$

with the boundary conditions

$$\begin{aligned}
 T &= T_0 \text{ for } r = \infty \\
 T &= T_0 \text{ and } n = 1 \text{ for } t = 0 \text{ and } r \geq a \\
 T &= T_1 \text{ for } t = 0 \text{ and } r < a \\
 T_{a-0} &= T_{a+0} \text{ for } t > 0
 \end{aligned}$$

$$\left(\frac{\partial T}{\partial r} \right)_{r=a+0} = \frac{a\rho_t c_t}{2\lambda} \left(\frac{\partial T}{\partial t} \right)_{r=a}$$

The calculation has been carried out for platinum wires 0.015, 0.030, 0.060 and 0.120 mm in diameter surrounded by a composition with the same decomposing properties as mercury fulminate. The following numerical values have been used:

$T_0 = 293^\circ\text{K}$, $\rho_t = 21.4 \text{ g/cm}^3$, $c_t = 0.033 \text{ cal/g C}$, $\rho = 2.5 \text{ g/cm}^3$, $c = 0.1 \text{ cal/g C}$, $\lambda = 8.10 \cdot 10^{-5} \text{ cal/cm s C}$, $A = 10^{11.05} \text{ s}^{-1}$, $E = 25400 \text{ cal/mol}$, $Q = 400 \text{ cal/g}$. The values of A and E are, according to Vaughan and Phillips (3).

Whether the charge explodes or the heat supplied is led off from the wire before any considerable decomposition takes place must be decided from the temperature curves obtained.

Fig. 1 shows the temperature versus time in the explosive for $a = 15 \mu$ (0.015 mm), $r = 15.6 \mu$ and six equidistant initial temperatures between $T_1 = 420$ and 470°C . A rapid rise in temperature is obtained for $T_1 = 470^\circ\text{C}$ after 0.7–0.8 milliseconds. Fig. 2a shows the temperature versus time for $T_1 = 470^\circ\text{C}$ at 7 points in the explosive medium between $r = 15.6 \mu$ and $r = 19.2 \mu$ at an interval of 0.6μ . The explosion occurs after 0.8–0.9 msec. Fig. 2b reproduces the temperature according to fig. 2a as a function of the distance from the wire surface with the time as

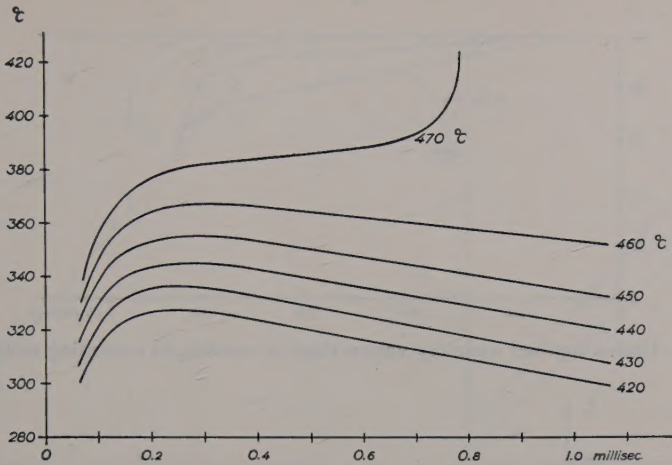


Fig. 1. Temperature versus time 0.6μ from the surface of a wire 30μ in diameter. The figures indicate the initial temperature of the wire.

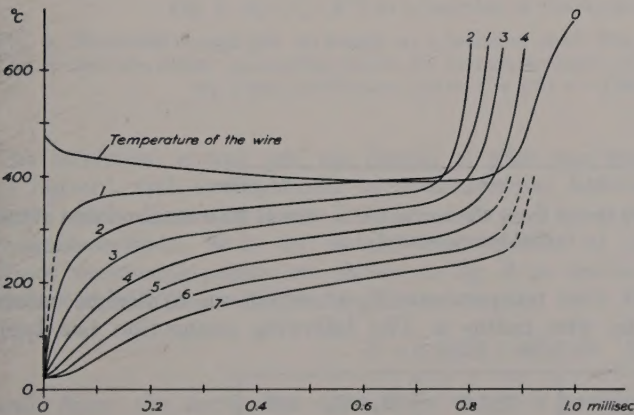


Fig. 2 a.

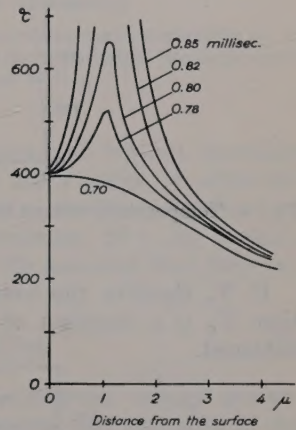


Fig. 2 b.

Fig. 2 a. Temperature versus time around a wire of 30μ diameter and 470°C initial temperature. The figures indicate rising radial distances with an interval of 0.6μ .

Fig. 2 b. Temperature versus the distance from the surface of the wire according to fig. 2 a.

parameter. Fig. 3 shows the undecomposed quantity as a function of the time under conditions according to fig. 2 a.

It is interesting to note that the temperature close to the wire surface after the first rapid rise can be practically constant for a long time before the temperature rise leading to explosion begins. Fig. 4 gives an example of the development of the temperature showing a distinctive "induction period". It is obvious that this is not a sign of any special delay in decomposition but is due to cooperation between the decomposition according to the Arrhenius function, the heat capacity and the heat transmission.

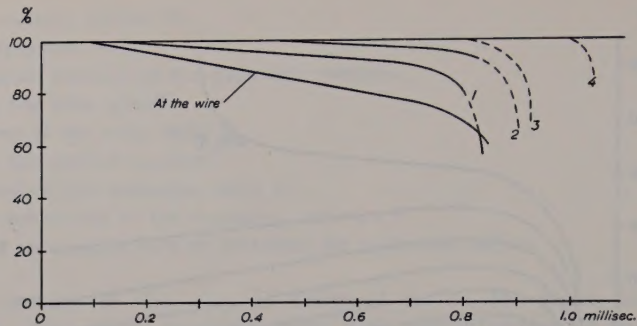


Fig. 3. Undecomposed quantity versus time on conditions according to fig. 2 a.

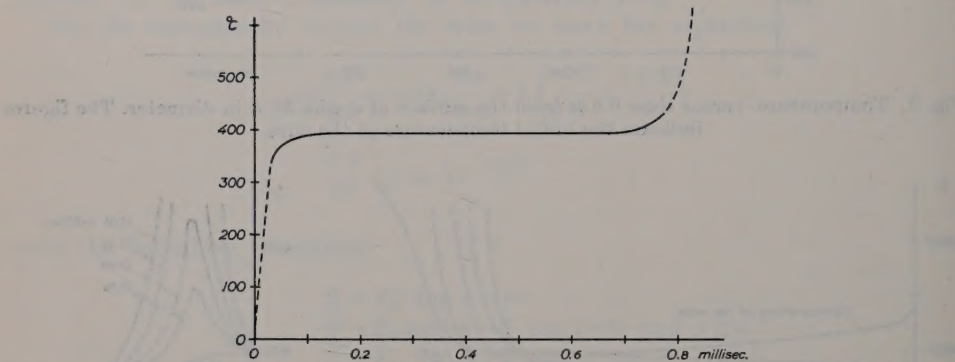


Fig. 4. Temperature versus time 0.4μ from the surface of a wire of 30μ diameter and 470°C initial temperature.

If T_e signifies the lowest wire temperature T_1 at which an explosion occurs then T_e is a function of the wire radius a . The following connection has been obtained.

a mm	$T_e^\circ\text{K}$	$\theta_e = T_e - 273$
0.06	683	410
0.03	703	430
0.015	743	470
0.0075	823	550

$\theta_e = T_e - 273$ is dotted in degrees Centigrade as a function of $1/a$ in fig. 5. The diagram shows that the dots lie on the straight line $\theta_e = 390 + 0.12/a$. According to this the explosion occurs when the energy supplied to the wire per cm length exceeds

$$U = 1.11 a + 3437 a^2 \quad \text{Ws/cm} \quad (1)$$

The continuous line in fig. 6 reproduces U as a function of a^2 according to equation 1.

The conditions for ignition by hot wires have been investigated by Jones (4).

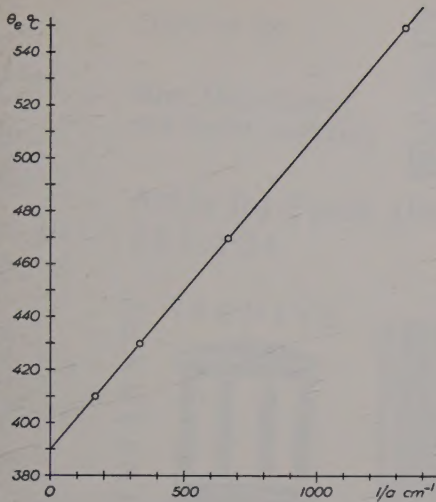


Fig. 5.

Fig. 5. $\theta_e = T_e - 273$ as a function of the inverted wire radius.

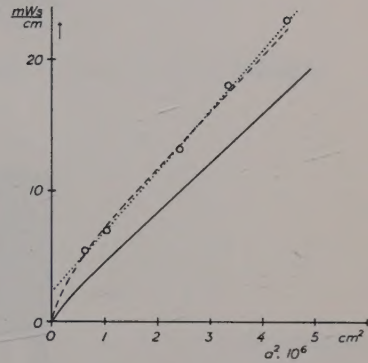


Fig. 6.

Fig. 6. Minimum energy per cm length as a function of a^2 . The unbroken curve represents eq. 1. The circlets are Jones' measuring points for monoresorcinate. The dotted straight line represents eq. 2 and the broken curve $U = 3.81 a + 3190 a^2$ Ws/cm.

The minimum energy per cm length of wire was measured at various strengths of current and extrapolated to instantaneous heating for a series of nichrome wires with diameters from 1.26 to 4.22 μ . The explosive medium consisted of monoresorcinate, 80 % by weight and chlorate of potassium, 20 % by weight. Jones measuring-points are shown in fig. 6 as circlets. He assumed that they lay on a straight line with the equation

$$U = 0.0022 + 4630 a^2 \quad \text{Ws/cm} \quad (2)$$

and drew the conclusion that there exists a positive minimum energy characteristic of the explosive, below which ignition is impossible. The accuracy of the measurements is not sufficient, however, to decide if the functional connection for U is of the type in equation 1 or 2. It seems to us more probable that U tends to zero with the radius.

LITERATURE

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